CAUSALITY: A FUTURE DIRECTION FOR MATHEMATICS OF UNCERTAINTY

JOHN N. MORDESON

Department of Mathematics
Creighton University
Omaha, Nebraska 68178 USA
mordes@creighton.edu

TERRY D. CLARK

Department of Political Science
Creighton University
Omaha, Nebraska 68178 USA
tclark@creighton.edu

BEVERLY DOYLE

Department of Education
Creighton University
Omaha, Nebraska 68178 USA
bdoyle@creighton.edu

LYNN OLSON

Department of Education
Creighton University
Omaha, Nebraska 68178 USA
lolson@creighton.edu

In this paper, we present techniques involving mathematics of uncertainty that can be applied to several new research areas. Two of the areas are children with special needs and nuclear stability. Among the techniques given here are the Guiasu method, the analytic hierarchy process, Dempster-Shafer theory, and fuzzy preferences relations. The techniques are used to measure degrees of causality.

Keywords: mathematics of uncertainty; causality, measurement; children with special needs; nuclear stability; Guiasu method; Dempster-Shafer theory; analytic hierarchy process; minimal disagreement approach; data envelopment analysis; fuzzy preference relations.

1. Introduction

Albert Einstein stated that development of Western science is based on two great achievements: the invention of the formal logical system and the discovery of the possibility to determine causal relationships by systematic experiment, Ref.1. In Ref. 1, Judea Pearl states that in the last decade (the 1990s), owing partly to advances in graphical models, causality has undergone a major transformation. Practical problems relying on causal information that long were regarded as unmanageable can now be solved using elementary mathematics. Pearl stresses that basic concepts of probability theory and graph theory is all that is needed for one to begin solving causal problems that are too complex for the unaided intellect.

We support the notion that problems of causality can be studied profitably by the use of mathematics of uncertainty and basic concepts from graph theory. We propose a model consisting of an overarching goal and components making up the overarching goal. We consider the components as nodes with a directed edges emanating from them to the overarching goal. We next consider attributes making up the components as nodes with directed edges emanating from them to the components. Each attribute has at most one edge directed to a particular component, but it may have a directed edge to more than one component. The resulting figure is a directed graph. The importance of each component and each attribute is weighted by means of expert opinion, data, and various mathematical techniques. The weights can be used to determine linear equations, where the overarching goal is the dependent variable and the components the independent variables. Each component can also be the dependent variable for a linear equation of the attributes as the independent variables. By proper substitution, the overarching goal can be written in terms of a linear equation involving the attributes. These linear equations give a measure as to how well the goals are being achieved at a specific time. The independent variables can be considered as causal variables. If directed edges are drawn between attributes to denote a weighted relationship between them, then a resulting mathematical structure is determined and is known as a directed graph or network. This structure can be studied in its own right.

In this paper, we illustrate this model by focusing on several new application areas, e.g., issues involving children with special needs and nuclear stability. Research on the application areas listed below are currently in progress. Students at Creighton University working on these projects are listed next to each application area. In the conclusion, we list the faculty at Creighton and staff from various institutions who are also involved.

Children with Special Needs (Michael Redmond)

There are about 7 million students with disabilities in public schools. Learning disabilities such as dyslexia are the most common, followed by speech or language impairments. Mental retardation and emotional disturbances rank next in frequency, though they show up in much smaller numbers.

The specific focus for deaf and hard of hearing children is on the development of the progression of auditory (listening) skills and the development of understanding and expression of spoken language. The goal of auditory and language instruction is to provide the children with the appropriate language stimulation and practice so the gap will continually close between their chronological age and their language age. The hope is that the children will learn to communicate effectively with their families, their classmates, their hearing peers, and to develop strategies for accessing information to learn in a regular classroom.

Language Environment Analysis (LENA) is a new recording device used in a parent - infant setting with the goals of (1) children making more progress in both receptive and expressive language, (2) professionals having more information, and (3) making parents more effective.

We consider two overarching goals for deaf and hard of hearing prekindergarten children : (1) which tests are the best predictors of success for children when mainstreamed in the public school system and (2) measuring how well children are closing the gap with respect to language growth. For either one of these goals, experts from Omaha Hearing School weighted fifteen test which they thought reflected the importance as indicating the success of obtaining the overarching goal. These weights were converted to numbers from the closed interval [0,1] using the Guiasu method and the analytic hierarchy process. This allowed the overarching goal to be expressed as a linear combination of the tests (the independent variables) with the coefficients of the independent variables the weights from [0,1] and whose sum equals 1.Test scores were coded using numbers from the closed interval [0,1]. These coded numbers can then be substituted into the linear equation to obtain a number from the interval [0,1] that expresses the success of how well the overarching goal is being achieved, Refs. 2, 3.

These ideas and techniques can be applied to measure progress of children with other special needs. Autism is one of five disorders coming under the umbrella of Pervasive Development Disorders, a category of brain disorders that affect the way people speak, socialize, play and react to their environment. There is no cure for autism and no single treatment package for all children with autism. However most professionals agree that early treatment is important and most people with autism respond well to highly structured specialized programs. The techniques described in this paper can be used to measure the success of many of these treatments. Many of these techniques are based on fuzzy set theory, Ref. 4.

Nuclear Stability (Hilary Wething)

An official statement from the Obama Administration stated that the gravest danger to the American people is the threat of a terrorist attack with a nuclear weapon and the spread of nuclear weapons to dangerous regimes. In this paper, we propose the use of fuzzy logic to model the bigger problem of nuclear stability. We base our work on the opinion of experts from government, military, industry, academia, and the literature, Refs. 5-17.

Nuclear stability can be defined as a condition of the international systems in which the actors are disinclined to acquire, proliferate, or use nuclear weapons. Nuclear stability is determined by various factors including: (1) programs to acquire, deploy, proliferate, threaten to use, or use fissile nuclear materials and/or weapons, (2) the security and control regimes of nuclear weapons programs, stockpiles, and fissile materials, (3) state and non-state programs to traffic illegally in nuclear weapons, delivery systems, and/or enabling technologies and materials.

Nuclear stability is made up of four main components: (1) Acquire, (2) Proliferate, (3) Use, and (4) Cooperate. Each component is made up of numerous attributes, namely, (1) enduring rivalries, (2) strategic culture, (3) international openness, (4) political stability, (5) international opinion, (6) credible security guarantee, (7) human capitol, (8) financial resources, (9) economic freedom, (10) globalization, (11) economic stability, (12) distribution networks, (13) delivery systems, (14) C2 (command and control) capability, (15) nuclear infrastructure, (16) threats to state survival, and (17) military stability.

We propose a model with nuclear stability as the overarching goal. We consider the four main components as nodes with a directed edges emanating from them to the nuclear stability node. We next consider attributes making up the components as nodes with directed edges emanating from them to the components. Each attribute has at most one edge directed to a particular component, but it may have a directed edge to more than one component. The resulting figure is a directed graph. The importance of each component and each attribute is weighted by means of expert opinion, data, and various mathematical techniques. The weights will be used to determine linear equations, where nuclear stability is the dependent variable and the four components the independent variables. Each component will also be the dependent variable for a linear equation of the attributes as the independent variables. By proper substitution the overarching goal, nuclear stability, can be written in terms of a linear equation involving the attributes. These linear equations give a measure as to how well the goals are being achieved at a specific time.

Economic Freedom (Hilary Wething)

Economic freedom is the fundamental right of every human to control his or her own labor and property. In an economically free society, individuals are free to work, produce, consume, and invest in any way they please, with that freedom protected by the state and unconstrained by the state. The Wall Street Journal and the Heritage Foundation have tracked the economic freedom of countries for fifteen years, Ref. 18. They have developed an Index of Economic Freedom. The index is based upon analysis of 10 specific components of economic freedom: (1) business freedom, (2) trade freedom, (3) fiscal freedom, (4) government size, (5) monetary freedom, (6) investment freedom, (7) financial freedom, (8) property rights, (9) freedom from corruption, and (10) labor freedom. The 10 composite scores are equally weighted and averaged to get an overall economic freedom score for each country. We propose several weighting systems based on fuzzy mathematics to determine an overall score.

Smart Power (Carly Goodman, Alex Pham)

The Center for Strategic and International Studies formed a bipartisan Commission on Smart Power to develop a vision to guide America's global engagement, Ref. 19. The Commission determined that the United States should concentrate on the following five areas: (1) alliances, partnerships, and institutions, (2) global development, (3) public diplomacy, (4) economic integration, and (5) technology and innovation. These five areas can be considered to be the components making up the overarching goal of improving the image of the United States. Techniques described above can be used to measure the effectiveness of the United States in achieving this goal.

Political Stability (Michael Gibilisco)

Some researchers have defined political stability as a state's ability to avoid revolutionary wars, ethnolinguistic wars, adverse regime changes, genocides and politicides, Refs. 20, 21. With this definition in mind, the overarching goal of political stability can be considered to be made up of the components, (1) economic strength, (2) government legitimacy, (3) institutional stability, and (4) strong external relations. Each of these components has numerous causal factors.

Cooperative Threat Reduction (Alex Pham)

The Defense Threat Reduction Agency program pursues four objectives to reduce the present threat of weapons of mass destruction and guarantee national security, namely, (1) dismantle former Soviet Union (FSU) weapons of mass destruction (WMD) and associated infrastructure, (2) consolidate and secure FSU WMD and related technology and materials, (3) increase transparency and encourage higher standards of conduct, and (4) support defense and military cooperation with the objective of preventing proliferation. Each of these objectives has numerous components, Ref. 22.

Failed States (Alex Pham, Michael Redmond, Michelle Garner)

The Fund for Peace has developed a Failed States Index, Ref. 23. The index is based on 12 factors: (1) demographic pressures, (2) refugees or internally displaced persons, (3) group grievance, (4) human flight, (5) uneven economic development, (6) economic decline, (7) delegitimization of the state, (8) public services, (9) human rights, (10) security apparatus, (11) factionalized elites, and (12) external intervention. The 12 composite scores are equally weighted and averaged to get an overall economic freedom score for each country. We propose several weighting systems based on fuzzy mathematics to determine an overall score.

Economic Stability (Chris Carlson)

Economic stability is made up of the 8 components, (1) low unemployment rate, (2) low inflation rate, (3) little volatility in economic activity, (4) little volatility in exchange rates, (5) little volatility in financial markets, (6) supportive governance practices, (7) increasing productivity, and (8) increasing efficiency, Ref. 24.

Creative Economy (Hilary Wething)

Creative economy or creative industries refers to a collection of interlocking industry sectors, Ref. 25. The UK Government Department for Culture, Media and Sport (DCMS) has produced the following definition of the creative industries: those industries which have their origin in individual creativity, skill and talent and which have a potential for wealth and job creation through the generation and exploitation of intellectual property. The DCMS definition recognizes 11 creative sectors: (1) advertising, (2) architecture, (3) arts and antique markets, (4) crafts, (5) design, (6) designer fashion, (7) film, video, and photography, (8) software, computer games and electronic publishing, (9) music and the visual and performing arts, (10) publishing, (11) television and radio.

Quality of Life (Alex Pham, Chelsea Fischer, Michael Davidson)

The Economist Intelligence Unit, an arm of the Economist Group which publishes The Economist newspaper, periodically collects data from 111 countries and publishes a Quality of Life Index. The nine quality-of-life factors are as follows: (1) material well-being, (2) health, (3) political stability and security, (4) family life, (5) community life, (6) climate and geography, (7) job security, (8) political freedom, (9) gender equality, Ref. 26.

Solar Energy (Michael Davidson)

Renewable energy sources are becoming increasingly popular. They are advantageous over traditional fossil fuels for a number of reasons including independence from unstable, foreign markets and reduction in environmental impact. Photovoltaic energy (PVE), more commonly known as solar power, is an especially promising energy resource. The overarching goal is the proliferation of PVE. The four sub-factors are (1) photovoltaic systems, (2) research and development of PV technologies, (3) economics, and (4) public perception, Ref. 27.

College Freshman Weight Gain (Chelsea Fischer)

As obesity becomes an ever increasing problem across the United States the impact caused by the epidemic is causing major health concerns. Some health conditions that are associated with obesity include cancer, type 2 diabetes, hypertension, stroke, sleep apnea and dyslipidemia. The transition from high school to college is well known for being a time of weight gain in a large portion of the population. This is of particular importance for colleges and universities at which freshmen live on campus and consume food in dining halls. As a result, one area that colleges and universities need to focus on their in their orientation and throughout the first year education is maintenance of a healthy life-style including prevention of weight gain. The prevention of weight gain is the overarching goal and their are various goals that are needed to be addressed in order to reach this overarching goal: (1) food consumed by freshman students, (2) the level of physical activity and exercise, (4) time constraints put on a college student; (5) the level of stress a college student is under, and (6) the social influences. There are a number of factors that have an impact on each goal, Refs. 28-33.

Population Management of Sub-Saharan Africa (Chelsea Mann)

The format includes an organized arrangement of factors that influence population processes, population management policies for the Sub-Saharan region and for each selected country, as well as social, cultural, ethical, and religious issues that shape their current population policies. Three factors making up the overarching goal are (1) processes, (2) management, and (3) social/cultural. Each factor is made up of several subfactors, Refs. 34-38.

Safe Skies (Michelle Garner)

September 11, 2001 opened the world's eyes to a new weapon of mass destruction; commercial airlines. We now live in an age where getting on an airplane requires as much background check and security as getting into the

White House. The overarching goal is keeping commercial airlines from harm. The factors to achieve this goal are (1) layered security, (2) ready response, (3) international cooperation, and (4) maximizing domain awareness, Refs. 39-40.

Denomination Splitting Among Protestants in the United States (Katherine Alexander)

Under the widespread umbrella of the term, Protestantism, is a major international religion with approximately 800 million Protestants worldwide. However, there are a diverse group with over 33,000 denominations in 238 countries. In the United States there are more than 6,000 denominations although these numbers are imprecise due to the ambiguity of the word "denomination." The factors causing splitting are (1) political differences, (2) charismatic issues, and (3) doctrinal differences, Refs.41-42.

Economic Growth in the United States (Chris Carlson)

Economic growth in the United States has fluctuated greatly since the 17th century. Over this time the United States has risen to become the largest economy in the world. Despite the continual growth the economy as been through periods of growth and recession determined by government policy, private sector factors, and communal influences. The factors making up economic growth are (1) government policy, (2) private sector, and (3) community, Refs. 43-50.

Remittances (Jacob Huju)

Remittances are a sum of money (or formerly) a quantity of an item transferred from one place to another. In the context of immigration, remittance usually means money sent to a home country from an immigrant living in a host country. There are three main factors making up remittances: (1) political, (2) social, and (3) economic, Refs. 51-58.

Health Care (Michael Redmond, Chelsea Mann, Jacob Huju)

The results of any adjustment to a complex system are frequently counterintuitive. All U.S. citizens and legal aliens should have access to affordable, minimally adequate health care. Health care reform would involve making heath care available to approximately 45 million additional people. This will involve additional costs. Minimally adequate health care will be based on evidence-based procedures and health economics. The overarching goals are to establish a funds flow model of the current health care delivery system and to determine the most cost-effective method of making affordable, minimally adequate health care available to all U.S. citizens and legal aliens, Ref. 59-68.

Nuclear Deterrence (John Herr, Carly Goodman)

The goal of deterrence is to prevent aggressive action or WMD use by ensuring that, in the mind of the potential adversary, the risks of the action outweigh the benefits, while taking into count the consequences of inaction. The capabilities needed for tailored deterrence involve the full range of military capabilities, presence, and cooperation, as well as diplomatic, informational, and economic instruments, Refs. 69, 70.

Developmental Disorders in Children (Patrick Kilcoyne)

Pervasive development disorders is a group of disorders characterized by delays in the development of multiple basic functions including socialization and communication, Refs. 71, 72. There are three main factors making up developmental disorders: (3) pervasive developmental disorders, (3) neurological disorders, and (4) learning disabilities.

Human Development (Alex Pham)

The Human development Index (HDI) is an index used to rank countries by level of "human development," which usually implies whether a country is developed, developing, or underdeveloped, Ref. 73. The HDI combines three dimensions: (1) life expectancy at birth, is an index of population health and longevity, (2) knowledge and education, as measured by the adult literacy ratio (with two-thirds weighting) and the combined primary, secondary, and tertiary gross enrollment ratio (with one-third weighting), (3) standard of living, as measured by the natural logarithm of gross domestic product per capita at purchasing power utility.

Child Development (Alex Pham)

The Save the Children Fund (SCF) is a charity registered in England and Whales, Ref. 74. Its purpose is to change the plight of children that are denied proper health care, food, education, and protection. SCI highlights the following three areas: (1) nutrition, (2) equitable development, and (3) women's education and empowerment.

We discuss in the following new methods that can be used to determine the coefficients of the independent variables of the previously mentioned linear equations.

2. Guiasu's Method

Guiasu's method describes the process of reaching a verdict by a probabilistic weighting of the expert opinions, which permits us to determine a numerical relationship between a goal and the factors making up the goal. The classical rules from decision theory proposed by Hooper, Dempster, Bayes, and Jeffrey are special cases of Guiasu's weighting process (see Ref. 75, p. 167). Guiasu addresses differences in expert opinion over the likelihood (probability) of something being true as well as the credibility of the claims (our belief in its truth).

A body of information induces a credibility (or probability) distribution m on $\mathcal{P}(X)$, the set of all subsets of X. That is, m is a function of $\mathcal{P}(X)$ into the closed interval $[0,1]$, written $m : \mathcal{P}(X) \to [0,1]$, such that $m(A) \geq 0 \forall A \in \mathcal{P}(X)$ and $\sum_{A\subseteq X} m(A) = 1$. The class of focal subsets of X corresponding to m is denoted by $\mathcal{F}(X;m) = \{A \mid A \subseteq X, m(A) > 0\}$. A pair of dependent bodies of information, say i and j, induce a joint credibility distribution, namely $m_{ij} : \mathcal{P}(X) \times \mathcal{P}(X) \to [0,1]$ such that $m_{ij}(A,B) \geq 0$ and

$$\sum_{A\subseteq X} \sum_{B\subseteq X} m_{ij}(A,B) = 1.$$

If the bodies of information are independent, then $m_{ij} = m_i m_j$. The corresponding class of focal pairs of subsets is $\mathcal{F}(X,X;m_{ij}) = \{(A,B) \mid A,B \subseteq$

$X, m_{ij}(A,B) > 0\}$. The weights corresponding to the body of information for which m is the credibility distribution are $w(\cdot \mid \cdot) : \mathcal{P}(X) \times \mathcal{F}(X;m) \to [0,\infty)$. The weighted body of information provides the new credibility distribution on $\mathcal{P}(X)$ given by

$$\mu(C) = \sum_{A \in F(X;m)} w(C \mid A) m(A).$$

We can generalize this procedure to formulate the way weights $w_{ij}(\cdot \mid \cdot,\cdot)$ are assigned to a mixed body of information inducing a joint credibility distribution induced on $\mathcal{P}(X)$. The weighted (i,j)-th body of information is

$$\mu_{ij}(C) = \sum_{(A,B) \in \mathcal{F}(X,X;m_{ij})} w_{ij}(C \mid A,B) m_{ij}(A,B), C \in \mathcal{P}(X),$$

where $w_{ij}(C \mid A,B)$ is the weight of the subset C given $(A,B) \in \mathcal{F}(X,X;m_{ij})$. This can, of course, be extended to any number of dimensions.

3. Belief and Plausibility Functions

Dempster Shafer theory was developed to deal with subjective probabilities in which probability measures one's degree of belief in a statement or hypothesis (that is, the credibility of the statement), Ref. 76. It is based on two dual nonadditive measures, namely, belief measures and plausibility measures. These measures can be defined in terms of a function $m : \mathcal{P}(X) \to [0,1]$ such that $m(\emptyset) = 0$, where $\emptyset$ denotes the empty set, and $\sum_{A \in \mathcal{P}(X)} m(A) = 1$. The function m is called a basic probability assignment. For all $A \in \mathcal{P}(X)$, $m(A)$ is the proportion to which all available and relevant evidence supports the claim that a particular element of X belongs to A. The number $m(A)$ pertains only to the set A. The value $m(B)$ for a subset B of A cannot be determined from A. For example, it is not required that $m(B) \leq m(A)$. It is also not required that $m(X) = 1$ or that there be a relation between $m(A)$ and $m(\overline{A})$. A belief function is defined in terms of m in the following manner: $Bel : \mathcal{P}(X) \to [0,1]$ such that $\forall B \in \mathcal{P}(X), Bel(B) = \sum_{A \mid A \subseteq B} m(A)$. The plausibility function is defined by $Pl(B) = \sum_{A \mid A \cap B \neq \emptyset} m(A)$. For all $A \in \mathcal{P}(X), Bel(A)$ is interpreted as the degree of belief based on available evidence that a given element of X belongs to A.

Evidence obtained in the same context from two independent sources and expressed by basic probability assignments m_1 and m_2 on $\mathcal{P}(X)$ can be combined to obtain a joint basic probability assignment m_{12}. We sketch the standard way of combining them. Let $K = \sum_{A \cap B = \emptyset} m_1(A) m_2(B)$. Suppose $K < 1$. Define $m_{12} : \mathcal{P}(X) \to [0,1]$ by $\forall C \in \mathcal{P}(X)$,

$$m_{12}(C) = \sum_{A \cap B = C} m_1(A) m_2(B)/[1-K],$$

if $C \neq \emptyset$ and $m_{12}(\emptyset) = 0$. Then the above formula is referred to as Dempster's rule of combination.

If the basic probability assignments are determined by expert opinion, say of factors making up a goal, then the belief of a subset of the set of all factors can be interpreted as the degree of importance of that subset in determining the overarching goal. Also, if the basic probability assignments are positive only on singleton sets, say for example single factors, then the combination of these basic probability assignments into a single basic probability assignment can be used as coefficients for the factors of the linear equation of the overarching goal expressed in terms of its factors.

Yen's method, Ref. 77, is a generalization of the Dempster-Shafer theory to allow for fuzzy subsets. It addresses the issue of managing imprecise and vague information in evidential reasoning by combining the Dempster-Shafer theory with fuzzy set theory. Several researchers have extended the Dempster-Shafer theory to deal with vague information, but their extensions did not preserve an important principle that the belief and the plausibility measures are lower and upper probabilities. Yen's method preserves this principle by employing normalized fuzzy measures of credibility. It also preserves the property that the belief of a (fuzzy) subset is the difference of one and the plausibility of the subset's complement. Further, it is more responsive to a change to a focal element's membership function than some approaches.

Yen's method involves various measures of subsethood and extends Dempster-Shafer's belief function by defining a measure of inclusion $I(A, B)$, the degree to which the fuzzy subset A is included in the fuzzy subset B by using the following formula,

$$Bel(B) = \sum_{A \in \mathcal{FP}(X)} I(A, B)m(A),$$

where $m : \mathcal{FP}(X) \to [0, 1]$ is such that $\sum_{A \in \mathcal{FP}(X)} m(A) = 1$ and where $\mathcal{FP}(X)$ denotes the set of all fuzzy subsets of X.

Let X be a nonempty set and A and B be fuzzy subsets of X. Let $\alpha \in [0, 1]$ and $A^\alpha = \{x \in X \mid A(x) \geq \alpha\}$. Then A^α is called an α-cut of A. Define $\overline{A} : X \to [0, 1]$ by $\forall x \in X, \overline{A}(x) = 1 - A(x)$. Then $\overline{A}$ is known as the standard complement of A. Let $I : \mathcal{FP}(X) \times \mathcal{FP}(X) \to [0, 1]$. Fuzzy set theory uses minimum to model intersection, represented with $\wedge$, and maximum to model union, represented by $\vee$.

There are a number of alternatives to Yen's method. They use measures of fuzzy subsethood. These approaches combine Dempster-Shafer theory and fuzzy set theory. The following are several measures of inclusion (the degree to which a fuzzy subset is a subset of another) (see Refs. 78-89): $\forall A, B \in \mathcal{FP}(X)$:

Bandler and Kohout:

$$I_{BK}(A, B) = \wedge\{\overline{A}(x) \vee B(x) \mid x \in X\},$$

Sanchez:

$$I_S(A, B) = \frac{\sum_{x \in X}(A(x) \wedge B(x))}{\sum_{x \in X} A(x),}$$

Wierman:

$$I_W(A,B) = \int_0^1 \chi_{\subseteq}(A^\alpha, B^\alpha)d\alpha,$$

where $\chi_{\subseteq}(A^\alpha, B^\alpha) = 1$ if $A^\alpha \subseteq B^\alpha$ and 0 otherwise.

4. Analytic Hierarchy Process

The Analytic Hierarchy process (AHP) provides the objective mathematics to express the subjective and personal preferences of an individual or a group in making a decision, Ref. 90. With the AHP and its generalization, the Analytic Network Process (ANP), one constructs hierarchies or feedback networks, then makes judgements or performs measurements on pairs of elements with respect to a controlling element to derive ratio scales that are then synthesized throughout the structure to select the best alternative.

Fundamentally, the AHP works by developing priorities for alternatives and the criteria used to judge the alternatives.

AHP is a systematic method for comparing a list of alternatives or objectives. Assume that a set of objectives has been established and that we wish to establish a normalized set of weights to be used when comparing alternatives using these objectives. We form a pairwise comparison matrix $A = [a_{ij}]$, where the number in the i-th row and j-th column gives the relative importance of objective O_i as compared with objective O_j. One may use a $1-9$ scale, where

$$a_{ij} = \begin{cases} 1\text{if } O_i \text{ and } O_j \text{ are of equal importance,} \\ 3 \text{ if } O_i \text{ is weakly more important than } O_j, \\ 5 \text{ if } O_i \text{ is strongly more important than } O_j, \\ 7 \text{ if } O_i \text{ is very strongly more important then } O_j, \\ 9 \text{ if } O_i \text{ is absolutely more important than } O_j, \end{cases}$$

and

$$a_{ij} = \begin{cases} 1\text{if } O_j \text{ and } O_i \text{ are of equal importance,} \\ 1/3 \text{ if } O_j \text{ is weakly more important than } O_i, \\ 1/5 \text{ if } O_j \text{ is strongly more important than } O_i, \\ 1/7 \text{ if } O_j \text{ is very strongly more important then } O_i, \\ 1/9 \text{ if } O_j \text{ is absolutely more important than } O_i, \end{cases}$$

The next step is to normalize the columns of A by computing the sum of each column and then divide each column by the corresponding sum. Ideally the normalized columns would all be identical if the pairwise comparisons were consistent. In practice, one can compute a consistency measure using the eigenvalues of the normalized matrix. The next step is to compute the average value of each row and use these values as the weights in what's called the Objective Hierarchy. These weights sum to 1. These weights would be used in summing the measures as required in the evaluation of the Objective Hierarchy.

The use of a $1-9$ scale is not the only approach. Experts may assign weights $w_i i = 1, ..., n$, of importance to the n priorities. Emphasis on consistency leads to the eigenvalue formulation $Aw = nw$, where n is the number of priorities and $w = (w_1, ..., w_n)$ is the vector consisting of the weights for the priorities.

Then the matrix of ratio comparisons $A = [w_i/w_j]$ is formed. Thus we have $Aw = nw$.

If a_{ij} represents the importance of alternative i over alternative j and a_{jk} represents the importance of alternative j over alternative k, the importance of alternative i over alternative k, must equal $a_{ij}a_{jk} = a_{ik}$ for the judgements to be consistent.

5. Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a data oriented approach for evaluating the performance of a set of peer entities, Refs. 91, 92. These entities are called Decision Making Units (DMUs). They convert multiple inputs into multiple outputs. There is a large variety of applications of DEA for use in evaluating the performance of many different types of entities. DEA requires very few assumptions. Consequently, possibilities have been opened for use in cases which have been resistant to other approaches due to the complex or unknown nature of the relations between the multiple inputs and multiple outputs in DMUs.

DEA is a fractional programming problem in linear programming methodology used when a direct causal relationship between inputs and outputs cannot be claimed. DEA measures the relative efficiency of these DMUs by finding the maximum of the ratio of the weighted outputs to inputs under the condition that the ratios of all the DMUs must be less than or equal to 1. Efficiency is described as the weighted sum of outputs to the weighted sum of inputs where the weights structure is figured out by mathematical programming. This weighting of "relative efficiency' is determined objectively to obtain a scalar measure of efficiency in any case. The DMU's are inefficiencies in inputs or outputs with "slack variables," and a DMU can be rated efficient if the maximum ratio is equal to 1 (100% return to scale) and the slack variables equal zero. There are four basic DEA models:

(1) the CCR ratio model,
(2) the BCC model,
(3) the Multiplicative model,
(4) the additive and the extended additive model.

The CCR model results in a piecewise linear, constant returns-to-scale envelopment surface. The BCC and Additive models yield in a piecewise linear, variable returns-to-scale envelopment surface. The Multiplicative models yield piecewise log-linear envelopment surfaces.

We sketch give a brief sketch of the "ratio-form" of DEA. Assume there are n DMUs to be evaluated. Each DMU consumes varying amounts of m different inputs to produce s different outputs. Specifically, DMU_j consumes amount x_{ij} of input i a produces amount y_{rj} of output r. We assume that $x_{ij} \geq 0$ and $y_{rj} \geq 0$. We also assume that each DMU has at least one positive input and one positive output value. The ratio of outputs to inputs is used to measure the relative efficiency of the DMU to be evaluated, say DMU_o, relative to the ratios of the DMU_j, $j = 1, 2, ..., n$. The CCR construction can be interpreted as the reduction of the multiple-output/multiple-input situation (for each DMU) to that of a single 'virtual' output and 'virtual' input. For a particular DMU the ratio of this single virtual output to single input provides a measure of efficiency that is a function of the multipliers. In mathematical programming terms, this ratio which is to be maximized, forms the objective function for the particular DMU being evaluated. That is,

$$\max h_o(\mathbf{u},\mathbf{v}) = \sum_{r=1}^{s} u_r y_{ro} / \sum_{i=1}^{m} v_i x_{io},$$

where $\mathbf{u} = (u_1, ...u_s)$ and $\mathbf{v} = (v_1, ..., v_m)$ and where the u_r and v_i are the variables and the y_{ro} and x_{io} are the observed output and input values, respectively, of the DMU_o to be evaluated. In order keep the solution from being unbounded, a set of normalizing constraints for each DMU is specified so that the ratio must be less than or equal to 1. The mathematical programming problem then becomes

$$\begin{aligned} \max h_o(\mathbf{u},\mathbf{v}) &= \sum_{r=1}^{s} u_r y_{ro} / \sum_{i=1}^{m} v_i x_{io}, \\ & \quad \text{subject to} \\ \sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij} &\leq 1 \text{ for } j = 1, ..., n, \\ u_r, v_i &\geq 0 \text{ for } r = 1, ..., s; i = 1, ..., m. \end{aligned}$$

6. Minimum Disagreement Approach

The Minimum Disagreement Approach (MDA) doesn't find a single best explanation, but a number of good explanations that satisfy pairwise independence, Ref. 93. The best hypothesis from a fixed set of alternative hypotheses is selected to determine a causal relationship. This is most frequently used with "noisy data," a term used in information extraction analysis.

We assume we have an overarching goal G supported by goals $G_i, i = 1, ..., n$, for which we wish to measure using k predetermined factors. Let Z_{ij} be the score of factor j for $Gi, j = 1, ..., k$. We impose the following rules for each G_i:

We assume the overall score G_i is a linear function of the Z_{ij}, i.e.,

$$G_i = \sum_{j=1}^{k} w_j Z_{ij},$$

where G_i is the overall score of goal i and w_j is the importance weight of factor $j, j = 1, ..., k$.

We require

$$\sum_{j=1}^{k} w_j = 1.$$

In order to insure each factor has an impact on G_i, we impose bounds on each w_j,i.e.,

$$b_j \geq w_j \geq c_j,$$

where the b_j and c_j are predetermined by the evaluator,$j = 1, ..., k$.

The problem becomes given the factor rating Z_{ij} how is a ranking of the G_i determined that minimizes the conflicts and disagreements among the G_i. For this model, there are two stages. First the set of weights for each G_i is calculated giving each G_i the best ranking. Second, the model aggregates ranks from all goals to resolve conflicts with minimum disagreement.

7. Fuzzy Preference Relations

Once the experts have provided their weights as to the importance of the factors, there are several techniques, Refs. 94-97, that can be applied to these weights to determine fuzzy preference relations on X, the set of factors. Let ρ_k denote the fuzzy preference relation for expert $k, k = 1, ..., n$. There are many techniques which can be used to aggregate these fuzzy preference relations into a single fuzzy preference relation ρ. Each of these fuzzy preference relations can be represented by an $m \times m$-matrix. Let R_k denote the matrix $[r_{ij}^k]$ associated with $\rho_k, k = 1, ..., m$ and R denote the matrix $[r_{ij}]$ associated with ρ. We propose that these matrices can be used to employ fuzzy linguistic quantifiers to represent the notion of a fuzzy majority. Some fuzzy logic-based calculi of linguistically quantified propositions have been proposed which can make it possible to handle fuzzy linguistic quantifiers. These calculi have been applied in order to introduce a fuzzy majority (represented by a fuzzy linguistic quantifier) into group decision making and consensus formation models and also an implemented Decision Support System for consensus reaching.

An example of a *linguistically quantified proposition* is "most experts are convinced" and may generally written as Qy's are F, where Q is a linguistic quantifier, e.g., most, Y is a set of experts with $y \in Y$, and F is a *property*, e.g., convinced. the problem becomes to find the truth of such statements. Two basic calculi may be employed Ref. 98 and Refs. 99, 100. We present a method proposed by Zadeh. In Ref. 99, a fuzzy linguistic quantifier Q is assumed to be a fuzzy subset. For instance, $Q =$ 'most' may be given as

$$\mu_Q(x) = \begin{cases} 1 \text{ for } x \geq .8, \\ 2x - .6 \text{ for } .3 < x < .8, \\ 0 \text{ for } x \leq .3. \end{cases}$$

We propose the use of a fuzzy majority represented by a linguistic quantifier to determine the most important factors using the R_k and R. The *core* is an appealing concept to use. The core is defined as a set, C, of undominated factors, i.e., those not defeated in pairwise comparisons by a strict majority $r \leq n$. That is, $C = \{Fj | \nexists F_i \text{ such that } r_{ij}^k > .5 \text{ for at least } r \text{ experts}\}$. The concept of a core and other concepts to follow in this section can be generalized by replacing .5 by some $\alpha \in (0, 1)$.

Let

$$h_{ij}^k = \begin{cases} 1 \text{ if } r_{ij}^k < .5, \\ 0 \text{ otherwise.} \end{cases}$$

where $i, j = 1, ..., m$ and $j = 1, ..., n$. thus $h_{ij}^k = 1$ if factor Fj defeats Fi and $h_{ij}^k = 0$ otherwise. Let H_k denote the matrix $[h_{ij}^k], k = 1, ..., n$. Then

$$h_j^k = \frac{1}{m-1} \sum_{i=1, i \neq j}^{m} h_{ij}^k$$

is the extent to which expert k is not against factor Fj. Thus $h_j = \frac{1}{n} \sum_{k=1}^{n} h_j^k$ is the extent to which all the experts are not against Fj and $\nu_Q^j = \mu_Q(h_j)$ is the extent Q individuals are not against Fj. The fuzzy Q-core is defined to be the fuzzy subset

$$C_Q = \nu_Q^1/F1 + ... + \nu_Q^m/Fm,$$

i.e., a fuzzy subset of factors that are not defeated by Q experts.

Now let R denote the matrix $[r_{ij}]$, where

$$r_{ij} = \begin{cases} \frac{1}{m} \sum_{k=1}^{m} a_{ij}^k \text{ if } i \neq j, \\ 0 \text{ otherwise,} \end{cases}$$

where

$$a_{ij}^k = \begin{cases} 1 \text{ if } r_{ij}^k > .5, \\ 0 \text{ otherwise.} \end{cases}$$

Let

$$g_{ij} = \begin{cases} 1 \text{ if } r_{ij} > .5, \\ 0 \text{ otherwise} \end{cases}$$

which expresses whether or not s_i defeats s_j. Then

$$g_i = \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} g_{ij}$$

is the mean degree to which option s_i is preferred to all other options. Let $z_{Qi} = \mu_Q(g_i)$, the extent to which s_i is preferred to Q other options. The fuzzy Q-consensus winner is defined to be the fuzzy subset W_Q of the set of options by

$$W_Q = z_Q^1/s_1 + ... + z_Q^n/s_n.$$

W_Q represents the fuzzy subset of options that preferred to Q other options.

Other concepts that can be defined and used are the *degree of consensus*, *relevance of options*, *relevance of a pair of options*, and the *importance of experts.* The *degree of agreement* of experts can also be measured.

The technique of preference modelling from outranking relations can be employed. Let $S, I, \succ, R$ and $\tilde{}$ be relations on X, where $x, y \in X$,

S : **Outranking relation**. The statement xSy means "x is not worse than y" and reflects the presence of arguments strong enough to support this assertion.

I : **Indifference relation**. The statement xIy means "x and y are indifferent, i.e., roughly equal, and reflects the presence of arguments strong enough to support both the assertions xSy and ySx.

$\succ$: **Preference relation** The statement $x \succ y$ means "x is preferred to y" and both the presence of arguments strong enough to support the assertion xSy and the absence of similar arguments to support the assertion ySx.

R : **Incomparability relation** The statement xRy means x and y are incomparable" and represents the absence of arguments strong enough to support at least one of the assertions xSy or ySx.

$\tilde{}$: **Non-preference relation The statement** $x\tilde{}y$ means "we cannot discriminate between and y, x and t are either incomparable or indifferent": it reflects the absence of arguments strong enough to support at least one of the two assertions $x \succ y$ or $y \succ x$.

The relations $S, I, \succ, R$, and $\tilde{}$ can be fuzzified in a natural way as follows: Let (N, T, V) be a de Morgan triple such that $\forall x \in [0,1], T(x, N(x)) = 0$, where N is a negation operator, T is a t-norm, and V is a s-conorm. Let $I, \succ_{N,T}, R_N$, and $\tilde{}_{N,T}$ be defined in terms of S as follows: $\forall x, y \in X$,

Indifference Index: $i(S)(x,y) = S(x,y) \wedge S(y,x)$;

Incomparability Index: $R_N(S)(x,y) = N(S(x,y)) \wedge N(S(y,x))$;

Preference Index: $\succ_{N,T} (E)(x,y) = T[S(x,y), N(S(x,y))]$;

Non-preference Index: $\tilde{}_{N,T}(S)(x,y) = (N(\succ_{N,T} (S)(x,y)) \wedge (N(\succ_{N,T} (S)(y,x))$.

8. Strengthening and Weakening Nodes in a Digraph

In a digraph, there are nodes which may be exercising a disruptive of divisive influence while others are helpful. The structure may be built from many different kinds of relationships, such as sociometric choice, communication, or power. It is useful to have a way of characterizing, in digraph terms, the amount to which nodes contribute to the connectedness of the digraph, Refs. 101, 102.

9. Maximal Flow

In our digraph, there is a natural flow from the attributes to the overarching goal. Assuming that our digraph is a transport network, a simple, directed graph with a vertex, the source, with no incoming edges, a vertex, the sink, with no outgoing edges, and weights on the edges, capacities, we can consider the standard problem of maximal flow.

One can consider the problem of maximum flow with fuzzy network arc capacities. There are results which can be considered equivalent to the classic Ford and Fulkerson theorem concerning the minimum cut and maximum flow value. One may also consider real-valued flows in a network with fuzzy capacity constraints. This approach uses the notion of a fuzzy number since the arc capacities will be fuzzy numbers.

10. Conclusions

In this paper, we present mathematical techniques involving mathematics of uncertainty that can be applied to a wide variety of new research areas. Those involved in these research projects are: Dr. M. J. Wierman (Computer Science), Dr. Fong (Statistics), Drs. M. and D. Mallenby (Economics, Statistics), Ms. M. Kelly, Ms. L. Burton, Ms. B. Walker, Ms. A. Bell (Omaha Hearing School), Sister M. Faltus and Ms. Cathy Hirchert (Madonna School).

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