
A Fuzzy Mathematical Model of Reducing AIDS- Related Deaths

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PAPER 9

By Patrick T. Bruck

Abstract:

We analyzed the importance of various factors in reducing AIDS-related deaths in Africa, North America, and Europe. This analysis was based on expert opinions, relevant research literature, and fuzzy mathematical models.

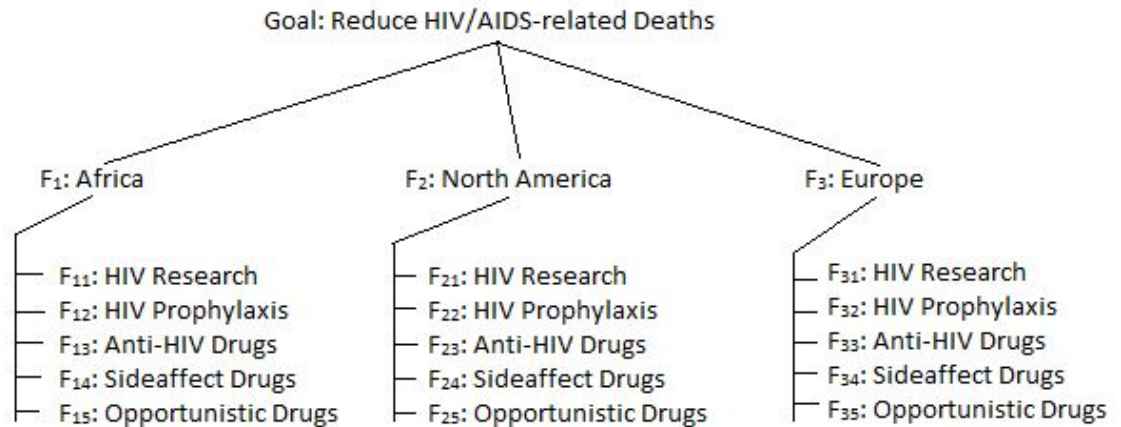
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1. Introduction

Human immunodeficiency virus-1 (HIV-1) is a major global issue responsible for more than thirty million deaths in the last thirty years [1]. HIV causes these deaths by killing the cells in the immune system of its host. In the end, HIV itself does not actually cause the death of its host. Instead, it drastically weakens their immune system resulting in a disease known as Acquired Immunodeficiency Syndrome (AIDS). Individuals with AIDS may then contract some other type of infection, oftentimes quite basic, which results in their death. The purpose of this paper is to analyze various factors in their importance for reducing AIDS-related deaths in Africa, North America, and Europe. These factors are defined as follows:

1. HIV research: research to better understand HIV itself, such as its life cycle & various functional mechanisms. Research need not be performed in the continent of interest to have an effect in that continent.
2. HIV prophylaxis: treatments to prevent HIV infection.
3. Anti-HIV drugs: antiretroviral drugs that treat HIV post-infection to reduce viral load.
4. Side effect drugs: drugs that treat the side effects of HIV as well as drugs that treat the side effects of other drugs that are treating for HIV.
5. Opportunistic drugs: drugs that treat opportunistic infections that take advantage of a weakened immune system.

We based our model off of the opinions of four experts in the field as well as relevant research literature. Each expert was asked to give a score of importance for each factor in reducing AIDS-related deaths within each of the continents of interest. Their rankings were then combined with data from the literature and analyzed using fuzzy mathematical methods to yield an order of significance for the factors. The factors are organized as shown in the tree below:



2. The Linear Equations

Analytical Hierarchy Process (AHP)

Each expert, E_n , has assigned a value to each subfactor as to its importance with respect to achieving the overarching goal G . In order to normalize the experts' data to form the root table, each expert's ranking for all subfactors of a continent were added together and divided by the total number that could be assigned for that continent, which was always 50. That number then represented that expert's ranking of that specific continent's importance in achieving the overarching goal. These data were compiled and analyzed using the AHP method [2, 3].

Root

Factor	E_1	E_2	E_3	E_4	Row Average
1. Africa	0.42	0.9	0.74	0.78	0.71
2. North America	0.68	0.82	0.8	0.78	0.77
3. Europe	0.7	0.82	0.8	0.78	0.775
Total	1.80	2.54	2.34	2.34	2.255

Africa

Factor	E_1	E_2	E_3	E_4	Row Average
1. HIV Research	0.3	0.7	0.6	0.9	0.6250
2. HIV Prophylaxis	0.4	1.0	0.9	0.9	0.8
3. Anti-HIV Drugs	0.3	1.0	0.9	0.8	0.75
4. Side effect Drugs	0.7	0.8	0.6	0.7	0.7
5. Opportunistic Drugs	0.4	1.0	0.7	0.6	0.675
Total	2.1	4.5	3.7	3.9	3.55

North America

Factor	E_1	E_2	E_3	E_4	Row Average
1. HIV Research	0.7	0.8	0.8	0.9	0.8

2. HIV Prophylaxis	0.6	0.8	0.8	0.8	0.75
3. Anti-HIV Drugs	0.6	1.0	0.9	0.9	0.85
4. Side effect Drugs	0.7	0.8	0.7	0.7	0.725
5. Opportunistic Drugs	0.8	0.7	0.8	0.6	0.725
Total	3.4	4.1	4.0	3.9	3.85

Europe

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.7	0.8	0.8	0.9	0.8
2. HIV Prophylaxis	0.6	0.8	0.8	0.8	0.75
3. Anti-HIV Drugs	0.7	1.0	0.9	0.9	0.875
4. Side effect Drugs	0.7	0.8	0.7	0.7	0.725
5. Opportunistic Drugs	0.8	0.7	0.8	0.6	0.725
Total	3.5	4.1	4.0	3.9	3.875

From the above tables, equations can be derived which compile the data and represent the importance of each factor or subfactor in achieving its respective goal. The values for the importance of each factor are obtained by dividing the row average of a certain factor by the row average of the sums of each column. Essentially, this divides the combined value for that goal by the total value of all goals to determine relative importance. These equations are shown below:

$$\text{Root: } G = (0.3149)(F_1) + (0.3415)(F_2) + (0.3437)(F_3)$$

$$\text{Africa: } G = (0.1761)(F_{11}) + (0.2254)(F_{12}) + (0.2113)(F_{13}) + (0.1972)(F_{14}) + (0.1901)(F_{15})$$

$$\text{North America: } G = (0.2079)(F_{21}) + (0.1948)(F_{22}) + (0.2208)(F_{23}) + (0.1883)(F_{24}) + (0.1883)(F_{25})$$

$$\text{Europe: } G = (0.2065)(F_{31}) + (0.1935)(F_{32}) + (0.2258)(F_{33}) + (0.1871)(F_{34}) + (0.1871)(F_{35})$$

Guiasu

Each column in the above AHP tables may be considered to be a probability distribution for each expert. The singleton sets consisting of a goal serve as the focal elements in the following model, termed the Guiasu model [4, Theorem 2.2., p. 122]. The Guiasu values may be found by dividing each expert's ranking for each factor by the sum total of all rankings for that expert.

Root

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. Africa	0.2333	0.3543	0.3162	0.3333	0.3093
2. North America	0.3778	0.3228	0.3149	0.3333	0.3372
3. Europe	0.3889	0.3228	0.3149	0.3333	0.34
Total	1.00	1.00	1.00	1.00	1

Africa

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
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1. HIV Research	0.1429	0.1556	0.1621	0.2308	0.1728
2. HIV Prophylaxis	0.1905	0.2222	0.2432	0.2308	0.2217
3. Anti-HIV Drugs	0.1429	0.2222	0.2432	0.2051	0.2034
4. Side effect Drugs	0.3333	0.1778	0.1621	0.1795	0.2132
5. Opportunistic Drugs	0.1905	0.2222	0.1892	0.1538	0.1889
Total	1	1	1	1	1

North America

Factor	E₁	E₂	E₃	E₄	Row Average
1. HIV Research	0.2059	0.1951	0.2	0.2308	0.2079
2. HIV Prophylaxis	0.1765	0.1951	0.2	0.2051	0.1942
3. Anti-HIV Drugs	0.1765	0.2439	0.225	0.2308	0.2190
4. Side effect Drugs	0.2059	0.1951	0.175	0.1795	0.1889
5. Opportunistic Drugs	0.2353	0.1707	0.2	0.1538	0.19
Total	1	1	1	1	1

Europe

Factor	E₁	E₂	E₃	E₄	Row Average
1. HIV Research	0.2	0.1951	0.2	0.2308	0.2065
2. HIV Prophylaxis	0.1714	0.1951	0.2	0.2051	0.1929
3. Anti-HIV Drugs	0.2	0.2439	0.225	0.2308	0.2249
4. Side effect Drugs	0.2	0.1951	0.175	0.1795	0.1874
5. Opportunistic Drugs	0.2286	0.1707	0.2	0.1538	0.1883
Total	1	1	1	1	1

These Guiasu tables yield the following equations when the row averages for each factor are divided by the row average of the column sums. Since the Guiasu results in a column sum of 1, and thus a column sum row average of 1, the Guiasu equations are essentially derived directly from the row averages. The Guiasu equations are shown below:

$$\text{Root: } G = (0.3093)(F_1) + (0.3372)(F_2) + (0.34)(F_3)$$

$$\text{Africa: } G = (0.1728)(F_{11}) + (0.2217)(F_{12}) + (0.2034)(F_{13}) + (0.2132)(F_{14}) + (0.1889)(F_{15})$$

$$\text{North America: } G = (0.2079)(F_{21}) + (0.1942)(F_{22}) + (0.2190)(F_{23}) + (0.1889)(F_{24}) + (0.19)(F_{25})$$

$$\text{Europe: } G = (0.2065)(F_{31}) + (0.1929)(F_{32}) + (0.2249)(F_{33}) + (0.1874)(F_{34}) + (0.1883)(F_{35})$$

Yen

To generate the Yen table [5], the largest element in each column is made equal to 1. Each entry in the column is then divided by the original value of the largest element. The row averages of the resulting matrix form a column vector which provides the weights for the factors.

Root

Factor	E₁	E₂	E₃	E₄	Row Average
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1. Africa	0.5999	1	1	1	0.9
2. North America	0.9715	0.9111	0.9959	1	0.9696
3. Europe	1	0.9111	0.9959	1	0.9768
Total	2.5714	2.8222	2.9918	3.0000	2.8464

Africa

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.4286	0.7	0.6667	1	0.6988
2. HIV Prophylaxis	0.5714	1	1	1	0.8929
3. Anti-HIV Drugs	0.4286	1	1	0.8889	0.8294
4. Side effect Drugs	1	0.8	0.6667	0.7778	0.8111
5. Opportunistic Drugs	0.5714	1	0.7778	0.6667	0.754
Total	3	4.5	4.1112	4.3334	3.9862

North America

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.875	0.8	0.8889	1	0.8375
2. HIV Prophylaxis	0.75	0.8	0.8889	0.8889	0.775
3. Anti-HIV Drugs	0.75	1	1	1	0.875
4. Side effect Drugs	0.875	0.8	0.7778	0.7778	0.8375
5. Opportunistic Drugs	1	0.7	0.8889	0.6667	0.85
Total	4.25	4.1	4.4445	4.3334	4.175

Europe

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.875	0.8	0.8889	1	0.8375
2. HIV Prophylaxis	0.75	0.8	0.8889	0.8889	0.775
3. Anti-HIV Drugs	0.875	1	1	1	0.9375
4. Side effect Drugs	0.875	0.8	0.7778	0.7778	0.8375
5. Opportunistic Drugs	1	0.7	0.8889	0.6667	0.85
Total	4.375	4.1	4.4445	4.3334	4.2375

These Yen tables yield the following equations when the row averages are divided by the row average of the column sums:

$$\text{Root: } G = (0.3327)(F_1) + (0.3337)(F_2) + (0.3337)(F_3)$$

$$\text{Africa: } G = (0.1753)(F_{11}) + (0.2240)(F_{12}) + (0.2081)(F_{13}) + (0.2035)(F_{14}) + (0.1891)(F_{15})$$

$$\text{North America: } G = (0.2081)(F_{21}) + (0.1943)(F_{22}) + (0.2189)(F_{23}) + (0.1886)(F_{24}) + (0.1901)(F_{25})$$

$$\text{Europe: } G = (0.2066)(F_{31}) + (0.1929)(F_{32}) + (0.2246)(F_{33}) + (0.1872)(F_{34}) + (0.1887)(F_{35})$$

3. Expert Influence and Opinion Change

As expert 4 was in a more influential position than the others, it is relevant to apply an influence equation in which that expert may have an effect on the opinions of the other experts. The equation is: $y_i^{(t+1)} = a_{ii} \sum_{j=1}^n [(w_{ij})(y_j^{(t)})] + (1-a_i)(y_i^{(0)})$, where y is an expert's ranking, t is time, i specifies which expert's opinion is under question, a_{ii} refers to that expert's susceptibility to influence, w_{ij} refers to expert j 's influence on expert i , y_j is expert j 's opinion at time t , and y_i is expert i 's opinion at time 0.

Using this equation, we found a second iteration, $t = 1$, which takes into account the influence and opinion changes exerted by expert 4. While calculating these values, experts 1, 2, and 3 were given influence values of 0.2222 while expert 4 was given influence values of 0.3333. This yielded the following set of data.

AHP

Root

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. Africa	0.558	0.758	0.7136	0.7259	0.6889
2. North America	0.558	0.7358	0.7358	0.7259	0.6889
3. Europe	0.558	0.7358	0.7358	0.7259	0.6889
Total	1.6741	2.2296	2.1852	2.1778	2.0667

Africa

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.5765	0.6654	0.6432	0.7370	0.6556
2. HIV Prophylaxis	0.7198	0.8531	0.8309	0.8407	0.8111
3. Anti-HIV Drugs	0.6543	0.8099	0.7877	0.7704	0.7556
4. Side effect Drugs	0.7000	0.7222	0.6778	0.7000	0.7000
5. Opportunistic Drugs	0.7407	0.6741	0.6444	0.6667	2.7259
Total	3.2580	3.7914	3.6136	3.6926	3.5889

North America

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.7864	0.8086	0.8086	0.8407	0.8111
2. HIV Prophylaxis	0.7210	0.7654	0.7654	0.7704	0.7556
3. Anti-HIV Drugs	0.7988	0.8877	0.8654	0.8704	0.8556
4. Side effect Drugs	0.7173	0.7395	0.7173	0.7148	0.7222
5. Opportunistic Drugs	0.7309	0.7086	0.7309	0.6741	0.7111
Total	3.7543	3.9099	3.8877	3.8704	3.8556

Europe

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.7864	0.8086	0.8086	0.8407	0.8111
2. HIV Prophylaxis	0.7210	0.7654	0.7654	0.7704	0.7556
3. Anti-HIV Drugs	0.8383	0.9049	0.8827	0.8852	0.8778
4. Side effect Drugs	0.7173	0.7395	0.7173	0.7148	0.7222

5. Opportunistic Drugs	0.7309	0.7086	0.7309	0.6741	0.7111
Total	3.7938	3.9272	3.9049	3.8852	3.8778

AHP equations at $t=1$:

$$\text{Root: } G = (0.3333)(F_1) + (0.3333)(F_2) + (0.3333)(F_3)$$

$$\text{Africa: } G = (0.1827)(F_{11}) + (0.2260)(F_{12}) + (0.2105)(F_{13}) + (0.1950)(F_{14}) + (0.1858)(F_{15})$$

$$\text{North America: } G = (0.2104)(F_{21}) + (0.1960)(F_{22}) + (0.2219)(F_{23}) + (0.1873)(F_{24}) + (0.1844)(F_{25})$$

$$\text{Europe: } G = (0.2092)(F_{31}) + (0.1948)(F_{32}) + (0.2264)(F_{33}) + (0.1862)(F_{34}) + (0.1834)(F_{35})$$

Guiasu

Root

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. Africa	0.3333	0.34	0.3266	0.3333	.3093
2. North America	0.3333	0.33	0.3367	0.3333	0.3372
3. Europe	0.3333	0.33	0.3367	0.3333	0.34
Total	1	1	1	1	1

Africa

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.1769	0.1755	0.1780	0.1996	0.1825
2. HIV Prophylaxis	0.2209	0.2250	0.2299	0.2277	0.2259
3. Anti-HIV Drugs	0.2008	0.2136	0.2180	0.2086	0.2103
4. Side effect Drugs	0.2149	0.1905	0.1876	0.1896	0.1956
5. Opportunistic Drugs	0.2273	0.1778	0.1783	0.1806	0.1857
Total	1	1	1	1	1

North America

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.2095	0.2068	0.2080	0.2172	0.2104
2. HIV Prophylaxis	0.1920	0.1958	0.1969	0.1990	0.1959
3. Anti-HIV Drugs	0.2128	0.2270	0.2226	0.2249	0.2218
4. Side effect Drugs	0.1911	0.1891	0.1845	0.1847	0.1873
5. Opportunistic Drugs	0.1947	0.1812	0.1880	0.1742	0.1845
Total	1	1	1	1	1

Europe

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.2073	0.2059	0.2071	0.2164	0.2092
2. HIV Prophylaxis	0.1900	0.1949	0.1961	0.1983	0.1948
3. Anti-HIV Drugs	0.2210	0.2304	0.2260	0.2278	0.2263
4. Side effect Drugs	0.1891	0.1883	0.1837	0.1840	0.1863

5. Opportunistic Drugs	0.1927	0.1804	0.1872	0.1735	0.1835
Total	1	1	1	1	1

Guiasu equations at $t=1$:

$$\text{Root: } G = (0.3333)(F_1) + (0.3333)(F_2) + (0.3333)(F_3)$$

$$\text{Africa: } G = (0.1825)(F_{11}) + (0.2259)(F_{12}) + (0.2103)(F_{13}) + (0.1956)(F_{14}) + (0.1857)(F_{15})$$

$$\text{North America: } G = (0.2104)(F_{21}) + (0.1959)(F_{22}) + (0.2218)(F_{23}) + (0.1873)(F_{24}) + (0.1845)(F_{25})$$

$$\text{Europe: } G = (0.2092)(F_{31}) + (0.1948)(F_{32}) + (0.2263)(F_{33}) + (0.1863)(F_{34}) + (0.1834)(F_{35})$$

Yen

Root

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. Africa	1	1	1	1	.3093
2. North America	1	0.9111	0.9959	1	0.3372
3. Europe	1	0.9111	0.9959	1	0.34
Total	3	2.8220	2.9918	3	0.9865

Africa

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.7783	0.7800	0.7742	0.8766	0.1825
2. HIV Prophylaxis	0.9718	1	1	1	0.2259
3. Anti-HIV Drugs	0.8834	0.9493	0.9482	0.9161	0.2103
4. Side effect Drugs	0.9454	0.8467	0.8160	0.8327	0.1956
5. Opportunistic Drugs	1	0.7902	0.7756	0.7931	0.1857
Total	4.5689	4.3662	4.3140	1	1

North America

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.9845	0.9082	0.9344	0.9658	0.9482
2. HIV Prophylaxis	0.9023	0.8599	0.8845	0.8848	0.8829
3. Anti-HIV Drugs	1	1	1	1	1
4. Side effect Drugs	0.8980	0.8305	0.8288	0.8221	0.8449
5. Opportunistic Drugs	0.9149	0.7958	0.8446	0.7745	0.8325
Total	4.6997	4.3944	4.4923	4.0915	3.5085

Europe

Factor	E ₁	E ₂	E ₃	E ₄	Row Average
1. HIV Research	0.9380	0.8937	0.9164	0.9500	0.9245
2. HIV Prophylaxis	0.8597	0.8459	0.8677	0.8705	0.8606
3. Anti-HIV Drugs	1	1	1	1	1
4. Side effect Drugs	0.8557	0.8173	0.8128	0.8077	0.8234

5. Opportunistic Drugs	0.8719	0.7830	0.8283	0.7616	0.8112
Total	4.5253	4.3399	4.4252	4.3898	4.4197

Yen equations at $t=1$:

$$\text{Root: } G = (0.3333)(F_1) + (0.3333)(F_2) + (0.3333)(F_3)$$

$$\text{Africa: } G = (0.1825)(F_{11}) + (0.2258)(F_{12}) + (0.2102)(F_{13}) + (0.1958)(F_{14}) + (0.1858)(F_{15})$$

$$\text{North America: } G = (0.2104)(F_{21}) + (0.1959)(F_{22}) + (0.2217)(F_{23}) + (0.1874)(F_{24}) + (0.1847)(F_{25})$$

$$\text{Europe: } G = (0.2092)(F_{31}) + (0.1948)(F_{32}) + (0.2263)(F_{33}) + (0.1863)(F_{34}) + (0.1835)(F_{35})$$

4. Results

At $t=0$, we used the equations derived from our tables to obtain the following results. The values substituted in for the factors were determined from the research literature in the field [6-11].

Root

	Africa	North America	Europe
AHP	2.873	2.1475	2.1517
Guiasu	2.8508	2.1475	2.1487
Yen	2.8595	2.1469	2.1488

Africa

	AHP	Guiasu	Yen
HIV Research	0.5283	0.5175	0.5247
HIV Prophylaxis	0.6759	0.6642	0.6705
Anti-HIV Drugs	0.7040	0.7100	0.6920
Side effect Drugs	0.3942	0.4062	0.4062
Opportunistic Drugs	0.5706	0.5661	0.5661

North America

	AHP	Guiasu	Yen
HIV Research	0.4851	0.4858	0.4858
HIV Prophylaxis	0.5192	0.5184	0.5184
Anti-HIV Drugs	0.5152	0.5117	0.5117
Side effect Drugs	0.3768	0.3780	0.3774
Opportunistic Drugs	0.2512	0.2536	0.2536

Europe

	AHP	Guiasu	Yen
HIV Research	0.4816	0.4823	0.4823
HIV Prophylaxis	0.5160	0.5152	0.5152
Anti-HIV Drugs	0.5271	0.5250	0.5243
Side effect Drugs	0.3774	0.3750	0.3750
Opportunistic Drugs	0.2496	0.2512	0.2520

At $t = 1$, an equation was applied to assert the potential influence of expert 4 on the opinions of the other experts. The new tables yielded new equations, which yielded the following results:

Root

	Africa	North America	Europe
AHP	2.8749	2.1515	2.1526
Guiasu	2.8742	2.1514	2.1519
Yen	2.8732	2.1518	2.1523

Africa

	AHP	Guiasu	Yen
HIV Research	0.5481	0.5472	0.5472
HIV Prophylaxis	0.6777	0.6777	0.6777
Anti-HIV Drugs	0.7020	0.7010	0.7000
Side effect Drugs	0.3900	0.3912	0.3912
Opportunistic Drugs	0.5571	0.5571	0.5571

North America

	AHP	Guiasu	Yen
HIV Research	0.4907	0.4907	0.4907
HIV Prophylaxis	0.5224	0.5224	0.5224
Anti-HIV Drugs	0.5180	0.5173	0.5173
Side effect Drugs	0.3744	0.3750	0.3750
Opportunistic Drugs	0.2460	0.2460	0.2464

Europe

	AHP	Guiasu	Yen
HIV Research	0.4879	0.4879	0.4879
HIV Prophylaxis	0.5192	0.5192	0.5192
Anti-HIV Drugs	0.5285	0.5278	0.5278
Side effect Drugs	0.3726	0.3726	0.3726
Opportunistic Drugs	0.2444	0.2444	0.2448

5. Discussion

The final data reveals that out of the three continents examined, the continent that is most important in reducing overall HIV/AIDS-related deaths is Africa. Africa appears to be about 25% more important than North America and Europe in achieving this overall goal. North America and Europe were essentially tied in their importance with respect to achieving the goal, with Europe scoring slightly higher.

Further insight into the data shows that within Africa, the most important tool for reducing HIV/AIDS-related deaths is anti-HIV drugs, followed closely by HIV prophylaxis. Of least importance in Africa are side effect drugs. In North America and Europe, we found a very similar, and much more even

distribution of importance among the factors. Anti-HIV drugs and HIV prophylaxis are again at the top. However, in North America and Europe opportunistic drugs scored the lowest.

After taking into account the influence a certain expert may have had, we computed a new set of data at $t = 1$. In this data, Africa is even slightly more in the lead, while North America and Europe are essentially tied with Europe again scoring slightly higher.

In Africa, we observed slight decreases in the importance of anti-HIV drugs, opportunistic drugs, and side effect drugs at the new time point. We also saw a slight increase in the importance of HIV prophylaxis and a more dramatic increase in HIV research.

In both North America and Europe, we observed slight decreases in the importance of side effect drugs and opportunistic drugs, but an increased emphasis on HIV research, HIV prophylaxis, and anti-HIV drugs.

Another interesting set of observations can be made by comparing the results produced by the various methods. Overall, the AHP, Guiasu, and Yen methods produced the same ranking of importance for factors and continents. There were a few points of disagreement, but these were generally caused by changes in thousandths of a decimal point in values that were already very close. Oftentimes, the scores given for a factor using the three different methods would produce numbers within a few ten-thousandths of a decimal point from each other. Almost always, factor values were within one hundredth of a decimal point from one method to the next.

In summary, we found that Africa should be our highest priority in reducing HIV/AIDS-related deaths. The most effective tools for doing so in Africa will be anti-HIV drugs and HIV prophylaxis. At the same time, North America and Europe are also quite important in achieving the overall goal. In these continents, the most effective tools for reducing HIV/AIDS-related deaths will also be anti-HIV drugs and HIV prophylaxis, but the other factors will also be of more importance in North America and Europe than in Africa. These assertions are subject to change over time, but it appears as if the change will be small and these factors will continue to remain of key importance for a significant period of time.

6. References

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